MATHS AND CLIMATE CHANGE: THE MELTING ARCTIC

2. Finding your way around the Arctic using GPS

- In a two-dimensional coordinate system mark the points P1 and P2 with coordinates (0, 20) and (30, 20) respectively. These denote the locations of two satellites that lie in a plane with the explorers (one unit in the coordinate system corresponds to 1000 km in our two-dimensional world, ground level lies on the *x*-axis). The receiver has calculated that the explorers are exactly 25 units (25,000km) away from each of the two satellites. Can you find the exact location of the explorers either by drawing circles centred on the two satellites or by using the equations of circles centred on the two satellites?

- Now consider the more general case, when the satellites and the explorers do not all lie in a plane. In three dimensions, what kind of shape is formed by all the points at distance d from the satellite at point P? Are two satellites enough to calculate the exact location of the explorers in three dimensions? How do questions 2 and 3 generalise to three dimensions?

- Try the remaining questions if you have time. (They are more challenging).

    1. Write down the equations of the sphere centred at the point (0,15,0) with radius 25, and the sphere centred at (0,-15,0) with radius 25. Observing the symmetry, what can you say about the circle of intersection?

    2. What are the centre and radius of the circle C of intersection?

    3. Observing the relative positions of the circle C of intersection and the sphere centred at the point (0,0,40) with radius 20, find the point(s) where the sphere and the circle meet.

- Conclusion: GPS systems are based on this process, which is called *trilateration*. However, to minimise errors and to gain extra information, real GPS systems calculate the distance between the receiver and four satellites.