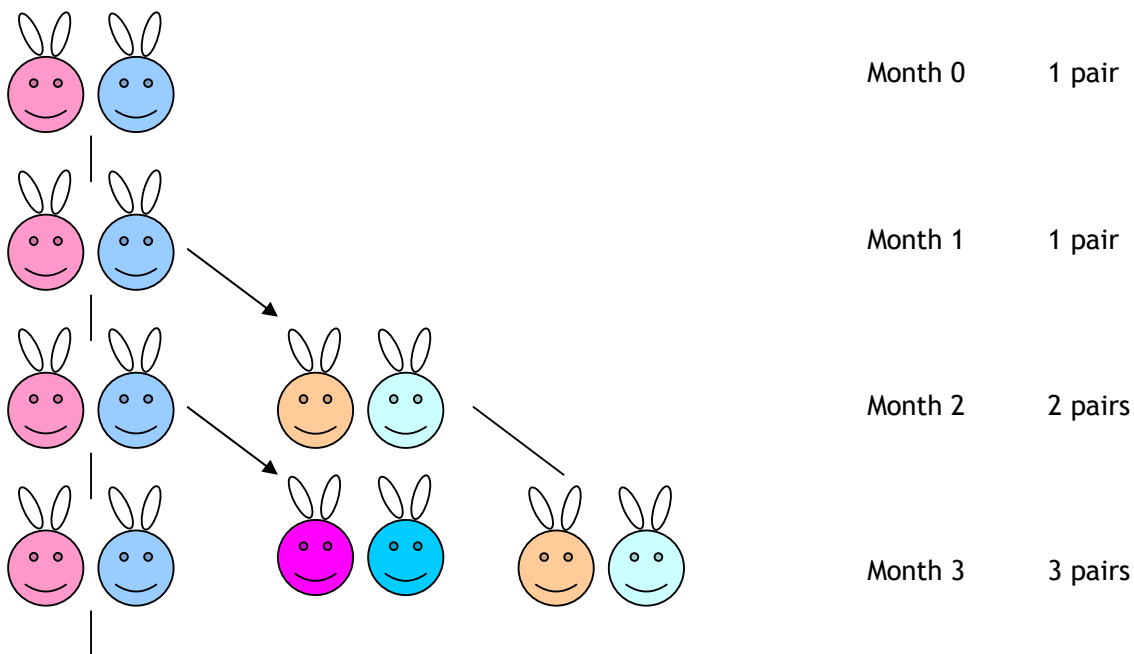


THE GOLDEN STRING

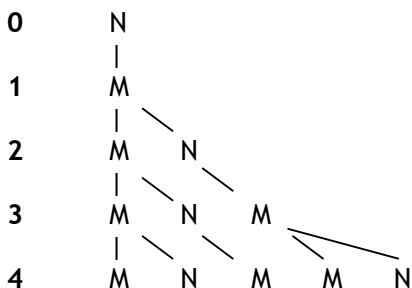
A problem often used to introduce the Fibonacci Sequence is the Rabbit Problem (because this is the problem Fibonacci, an Italian mathematician who lived from 1170 to 1250, used to introduce his sequence):

A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?

Copy and continue the diagram below for a year. Each pair of rabbits starts producing baby rabbits when it is 2 months old, and thereafter continues to produce another pair of baby rabbits every month (assume no rabbits die during this period!):



Month



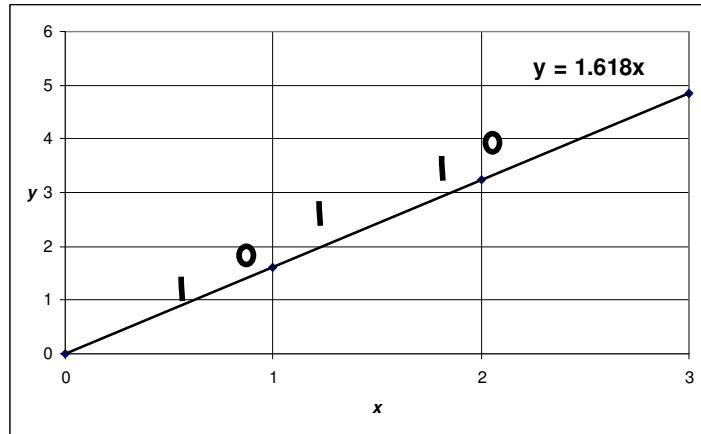
Now indicate each new pair with the letter N and each mature pair with the letter M, drawing up a diagram like that on the left. Note carefully the order - new offspring of a particular pair are shown before existing pairs are considered.

Can you find a rule to explain how to work out what the next month will be, given any previous month?

- What do you replace each M with?
- How about each N?

- Now instead of a sequence of Ns and Ms for each month, substitute 0 for N and 1 for M.

- Draw a graph of $y = 1.618x$ for $x \geq 0$ on squared paper with a scale of 1 unit for both x and y represented by 1 square on the paper (or use Excel, but be sure to set it so that it shows vertical and horizontal grid lines for each unit on the x - and y -axes). Take your values of x as high as you can for the piece of paper you are using.



- Wherever your line crosses a horizontal grid line on the paper, mark the point with a 1, and wherever your line crosses a vertical grid line on the paper, mark the point with a 0. Now write down the sequence of 1s and 0s you get, starting at the bottom left. What do you notice?

The string of 1s and 0s you get from either of these methods is called the Golden Sequence or Golden String. It is a *fractal* sequence, meaning that it is infinite and it contains copies of itself within itself.