

## MATHS AND MUSIC: DANCE MOVES (TEACHERS' NOTES AND ANSWERS)

Learning objectives:

- To revise/become familiar with the symmetries of the square
- To understand that some dance routines are basically just symmetries

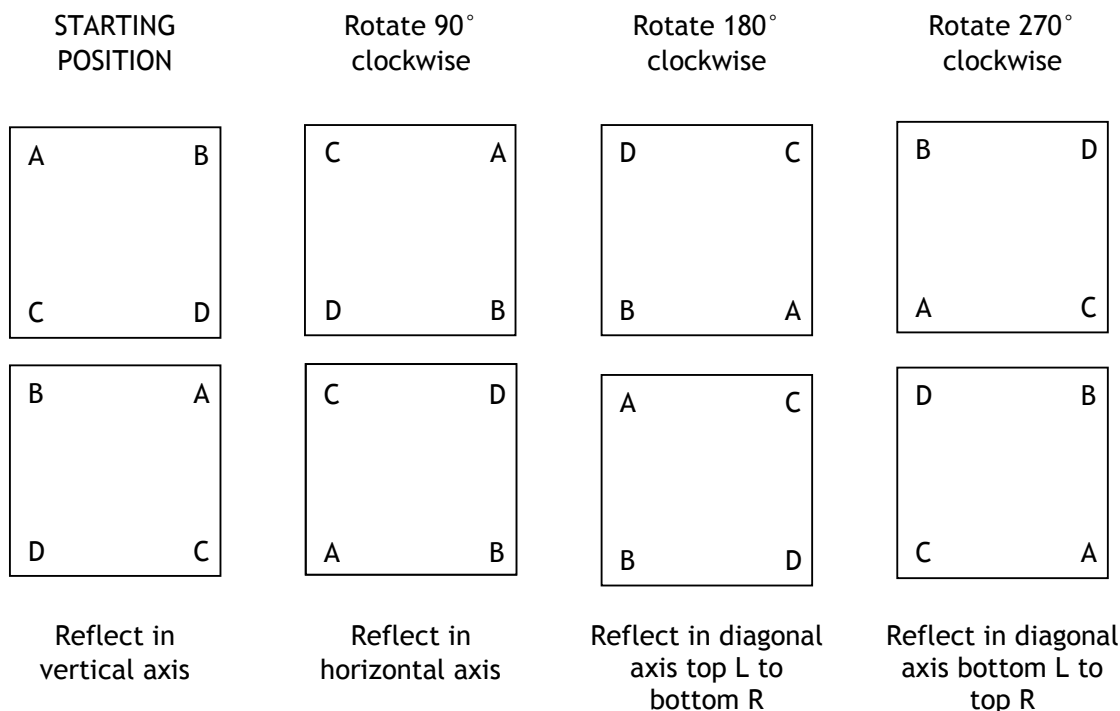
Students should check their answers at the smiley faces, making sure they understand any corrections.

*Students should work in small groups of at least 4 people throughout this activity. They will need to cut out the squares on the final sheet first.*

### Symmetries of the Square

Cut out the squares on the separate sheet and put one on the desk so it looks like the diagram on the top left below. This is the starting position each time. Use the second square to move as directed so you can directly compare the final position with the starting position.

1. Rotate the square through  $90^\circ$  clockwise (a quarter turn). It should match the labels in the second diagram now.
2. Rotate the square through  $180^\circ$  clockwise (a half turn) from the starting position. Label the 3<sup>rd</sup> diagram below to show where the labels end up.
3. Repeat for a rotation of  $270^\circ$  clockwise (a three quarter turn) from the starting position and label the 4<sup>th</sup> diagram below.
4. Put your square back into the starting position, then take a ruler and put it vertically down the centre of your square. If this is a mirror line, what happens to the labels? Check that you agree with the bottom left diagram below.
5. Keeping the square in the starting position, put the ruler horizontally across the middle of your square. If this is now a mirror line, show what happens to the labels in the appropriate diagram.
6. Repeat for reflections in the two diagonals of the square, with the square in the starting position each time.



These are the 8 symmetries of the square. In planning dance routines, we are going to use four of them:

- $e$  is the starting position
- $a$  is the rotation through  $90^\circ$  clockwise
- $b$  is the reflection in a diagonal axis from top L to bottom R
- $c$  is the vertical reflection

### Combining Symmetries

1. If you do first  $a$  and then, without going back to the starting position, follow it with  $b$ , which symmetry is this the same as? *Horizontal reflection*
2. If you do first  $b$  and then, without going back to the starting position, follow it with  $a$ , do you get the same result? *Vertical reflection =  $c$*
3. Do you find that surprising? *Not the same, so not like multiplication and addition (commutative) but like division and subtraction (not commutative)*
4. What happens if you do  $a$  four times? *Get back to the starting position =  $e$*
5. Find other combinations of  $a$ ,  $b$ ,  $c$  which get you back to  $e$ . *eg. a reflection followed by the same reflection, or two symmetries which make a reflection followed by the same two again, or a combination of rotations which amounts to a rotation through  $360^\circ$*



### Dance Moves

1. Inner twiddle:
  - Line up four people, labeled A, B, C and D respectively.
  - Swap the two middle people, so that the line-up is now A, C, B, D.
  - Convince yourselves that this is the same as the symmetry  $b$ .
2. Outer twiddle:
  - Line up four people, labeled A, B, C and D respectively.
  - Swap A with B and C with D, so that the line-up is now B, A, D, C.
  - Convince yourselves that this is the same as the symmetry  $c$ .
3. Combining twiddles:
  - Do an inner twiddle then an outer twiddle, without going back to the starting position.
  - Convince yourselves that this is the same as  $a$ , ie.  $b$  followed by  $c = a$
4. Combining more twiddles:
  - Repeat the inner-outer twiddle move four times, without going back to the starting position in between.
  - Have you now got back to the starting position?
  - What is this mathematically?

### Creating Dance Routines

- Find a mathematical sequence of symmetries of the square (not necessarily just  $a$ ,  $b$  or  $c$ ) and put them together. You should aim to start and finish in the starting position,  $e$ .
- Demonstrate your routines to each other, and show that they correspond to the mathematical sequence.

<p data-bbox="386 226 457 340"><b>A</b></p> <p data-bbox="1101 226 1172 340"><b>B</b></p> <p data-bbox="386 898 457 1012"><b>C</b></p> <p data-bbox="1101 898 1172 1012"><b>D</b></p>	<p data-bbox="386 1075 457 1188"><b>A</b></p> <p data-bbox="1101 1075 1172 1188"><b>B</b></p> <p data-bbox="386 1755 457 1869"><b>C</b></p> <p data-bbox="1101 1755 1172 1869"><b>D</b></p>
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