MATHS AND MUSIC: DANCE MOVES

When you reach a  check your answers, and make sure you correct any you got wrong.

Symmetries of the Square

Cut out the squares on the separate sheet and put one on the desk so it looks like the diagram on the top left below. This is the starting position each time. Use the second square to move as directed so you can directly compare the final position with the starting position.

1. Rotate the square through 90° clockwise (a quarter turn). It should match the labels in the second diagram now.
2. Rotate the square through 180° clockwise (a half turn) from the starting position. Label the 3rd diagram below to show where the labels end up.
3. Repeat for a rotation of 270° clockwise (a three quarter turn) from the starting position and label the 4th diagram below.
4. Put your square back into the starting position, then take a ruler and put it vertically down the centre of your square. If this is a mirror line, what happens to the labels? Check that you agree with the bottom left diagram below.
5. Keeping the square in the starting position, put the ruler horizontally across the middle of your square. If this is now a mirror line, show what happens to the labels in the appropriate diagram.
6. Repeat for reflections in the two diagonals of the square, with the square in the starting position each time.

STARTING POSITION	Rotate 90° clockwise	Rotate 180° clockwise	Rotate 270° clockwise
A B C D	C A D B		
B A D C			
Reflect in vertical axis	Reflect in horizontal axis	Reflect in diagonal axis top L to bottom R	Reflect in diagonal axis bottom L to top R

These are the 8 symmetries of the square. In planning dance routines, we are going to use four of them:

- *e* is the starting position
- *a* is the rotation through 90° clockwise
- *b* is the reflection in a diagonal axis from top L to bottom R
- *c* is the vertical reflection

Combining Symmetries

1. If you do first *a* and then, without going back to the starting position, follow it with *b*, which symmetry is this the same as?
2. If you do first *b* and then, without going back to the starting position, follow it with *a*, do you get the same result?
3. Do you find that surprising?
4. What happens if you do *a* four times?
5. Find other combinations of *a*, *b*, *c* which get you back to *e*.

Dance Moves

1. Inner twiddle:
- Line up four people, labeled A, B, C and D respectively.
- Swap the two middle people, so that the line-up is now A, C, B, D.
- Convince yourselves that this is the same as the symmetry *b*.

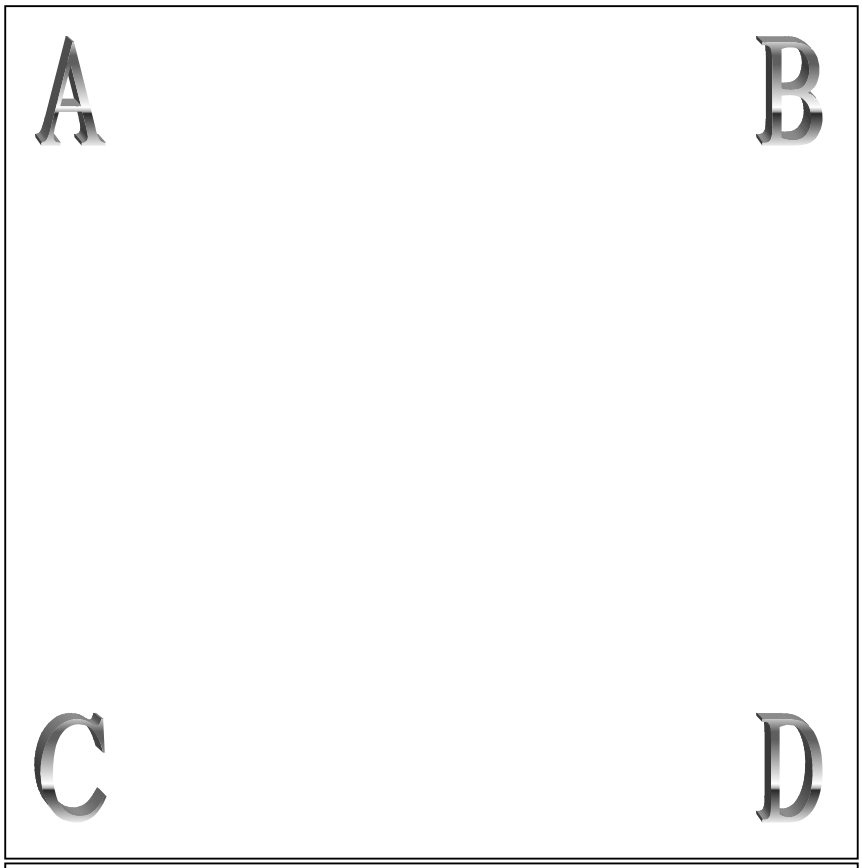
2. Outer twiddle:
- Line up four people, labeled A, B, C and D respectively.
- Swap A with B and C with D, so that the line-up is now B, A, D, C.
- Convince yourselves that this is the same as the symmetry *c*.

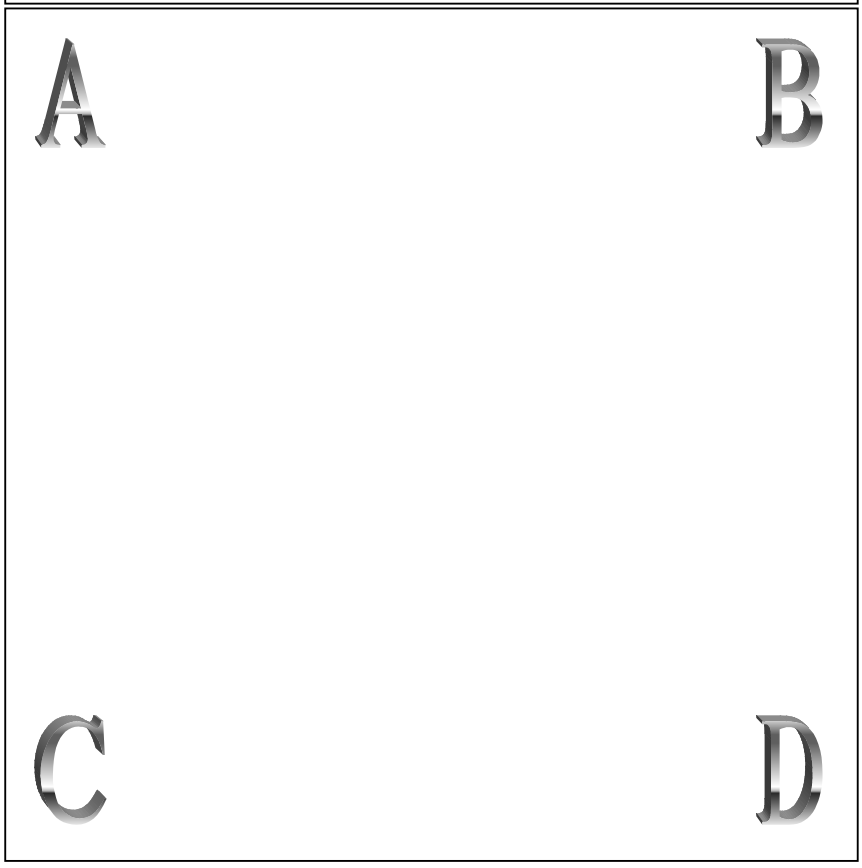
3. Combining twiddles:
- Do an inner twiddle then an outer twiddle, without going back to the starting position.
- Convince yourselves that this is the same as *a*, ie. *b* followed by *c* = *a*

4. Combining more twiddles:
- Repeat the inner-outer twiddle move four times, without going back to the starting position in between.
- Have you now got back to the starting position?
- What is this mathematically?

Creating Dance Routines

- Find a mathematical sequence of symmetries of the square (not necessarily just *a*, *b* or *c*) and put them together. You should aim to start and finish in the starting position, *e*.
- Demonstrate your routines to each other, and show that they correspond to the mathematical sequence.


A
B
C
D


A
B
C
D