Babylonian Maths

Teacher Notes and Worksheet Answers

Further information on Babylonian mathematics and associated topics can be found at

http://it.stlawu.edu/~dmelvill/mesomath/

The Babylonian number system

The Babylonians used a base 60 number system. This survived them, particularly in astronomy, where it was used for another thousand years after cuneiform (the style of their writing) died out under pressure from other alphabets, such as the Greek and Aramaic ones. The base 60 number system is still used for measuring time - 60 seconds in a minute, 60 minutes in an hour - and for measuring angle - 360 degrees in a full turn.

All Babylonian number symbols are in cuneiform script, and are formed by marks made by pressing a square-ended stylus into soft clay or similar.

- One mark corresponds to 1, two to 2, and so on. The pattern is to use rows of three, so 6 is given by two rows of three, 9 by three rows of three.
- Moving up from 9, a 10 was formed by turning the stylus through an angle of 30 to 45 degrees, giving a wedge shape. Again, multiples were formed in sets of three.
- All Babylonian numbers were formed in base 60, so there are 59 different symbols for their digits (as we have the nine symbols from 1 to 9). Once they got to 59, they used a place system to indicate this. The symbols used in, say, 23, together form the units in a base 60 number. 60 (base 10) corresponds to 1 00 (base 60), 190 (base 10) corresponds to 3 10 (base 60), where the gap indicates the distinction between units on the right and sixties on the left (so like columns for tens and units in the decimal system).
- There is no largest number in cuneiform - this system can be adapted for numbers as large as you need. The third place in a Babylonian number (equivalent to the hundreds column in a decimal number) was for 60 x 60 = 3600. 2 13 20 thus represents 2 x 3600 + 13 x 60 + 20 = 8000.
- The Babylonians usually represented zero with nothing - a blank space. Any zeros at the end of a number would have had to be remembered, and those in the middle of a number could easily have been missed on copying.

Babylonian fractions

Babylonian fractions also used base 60, so, for instance 0;15 corresponds to 15/60 or 0.25 and 3;45 corresponds to $3\frac{45}{60}$ or 3.75. In cuneiform, the zero in 0;15 would not have been shown - this is a modern convention. It would have been impossible to tell, just by looking at the notation 15, whether it meant 15 or 15 x 60 or 15/60, or any 15×60^n .

Because the denominator is not shown separately, it is always 60 for the first fractional place, 3600 (= 60^2) for the second, and so on. This means that many fractions - all those which cannot be expressed exactly in this way - do not have an exact equivalent which can be expressed in cuneiform. So, for instance, 1/7 does not have a cuneiform equivalent. The best that we can do is to write an approximation to such fractions. However because 60 has more factors than 10, there are more fractions which can be expressed finitely in base 60. So 2/3 = 0;40 in base 60, which is a finite fraction, whereas it is $0.\bar{6}$ in the decimal representation, which is non-terminating. All cuneiform fractions can be expressed in modern notation, however, since we can show a denominator as well as a numerator.

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The Resources

Below is the list as on the webpage for this pack. This suggests an order in which the resources might be used. Any resources can, of course, be used independently of the rest, but those that follow the video clips on Babylonian Numbers do require students to have seen them first.

* indicates a resource suitable for almost all students

** indicates a resource which requires students to work with place value in base 60 (following the video clips which explains how the Babylonian number system works) - which should help them to understand place value in base 10 better

*** indicates more challenging resources, involving multiplication, division and fractions in base 60 - again this should help them to understand these concepts better in base 10

Type of Resource	Resource Name	Difficulty	Notes
	Eleanor Robson: Maths Archaeologist	*	Introductory video clip (1 min 47 secs)
	Archaeology and your maths classroom	*	Video clip (2 mins 36 secs)
	Activity	*	If there was a fire or an earthquake tonight and your classroom was destroyed, what would a maths archaeologist find? What might s/he think about your maths class?
	A Babylonian House	*	Video clip (3 mins 20 secs)
	Babylonian Houses	**	Worksheet - do a scale drawing of a Babylonian house or see how the area of a Babylonian house compares with a modern one by finding rectangular areas. You will need to work in cubits to start with!
	Clay Tablets	*	Video clip (2 mins 56 secs)
	Make Your Own Clay Tablets	*	Presentation - make your own Babylonian tablet, complete with Babylonian numbers.
	Babylonian Triangles		Video clip (2 mins 19 secs)
	Triangles and Squares	*	Worksheet - area of squares and triangles (counting squares is fine for this), symmetry, investigation
	Babylonian Numbers: 1 to 9	**	Video clip (2 mins 36 secs)

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Type of Resource	Resource Name	Difficulty	Notes
	Babylonian Numbers: tens and units	**	Video clip (1 min 44 secs)
	Babylonian Numbers: base 60	**	Video clip (1 min 59 secs)
	Numbers in base 60	**	Presentation - working with numbers in base 60, helps with understanding of place value
	Numbers in base 60	**	Worksheet - to follow-up the presentation
	Fractions in base 60	***	Worksheet - this follows on from Numbers in base 60
	Learning to multiply - the Babylonian way	**	Video clip (3 mins 16 secs)
	Multiplication - the Babylonian way	**	Worksheet - students will need to know about numbers in base 60
	Division - the Babylonian way	***	Worksheet - students will need to know about multiplication and fractions in base 60
	Deciphering a Babylonian Tablet	***	Worksheet - drawings of multiplication problems, one involving simple fractions, on two Babylonian tablets to decipher. Did the Babylonian scribes get them right? It would help students to have seen the video clip Babylonian Triangles.

Worksheet Answers

Babylonian Houses

A modern 3-bedroomed house might have a ground floor area of between 50 and 90 sq m. But of course, there will also be first and perhaps second floor rooms. Babylonian houses were single storey.

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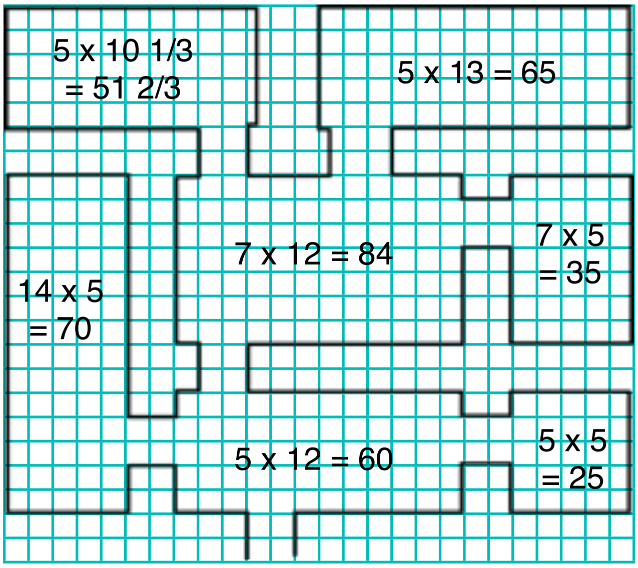


There is some estimation needed in making a scale drawing, but the room sizes can be determined without too much difficulty.

The total area is

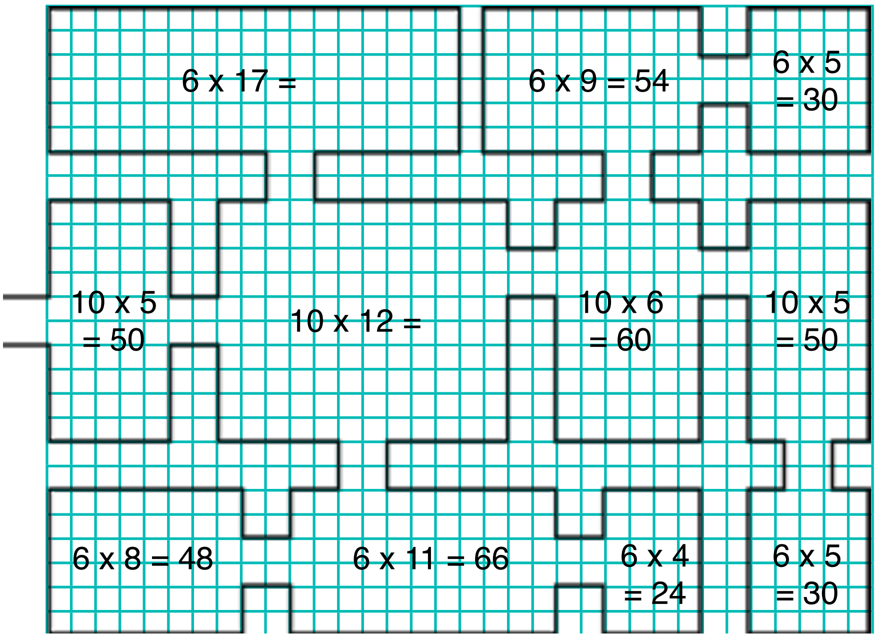
390 2/3 sq cubits or approx 97 2/3 sq metres.

It is important that students realise that if

1 cubit = 1/2 m then 1 sq cubit = 1/4 sq m.

For this house, the total area is

634 sq cubits or approx 158.5 sq m.



Triangles and Squares

The standard Babylonian unit of length was the cubit which was the length of a man's fore-arm, or about 50cm.

6 barley-corns = 1 finger

30 fingers = 1 cubit

6 cubits = 1 reed

12 cubits = a rod (so a rod is about 6m)

5 rods = 1 chain

10 rods = 1 rope

The area of a small square in the centre of the design and one of the triangles which surround these squares are the same. The area of the triangles immediately around the small squares and the corner triangles are the same. The exact number of area units will depend on the squared paper used and the scale of the drawing.

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The four small squares in the centre will have a length of 30 rods, so one small square has a length of 15 rods (about 90m). The area would then have been 225 square rods, and a triangle the same.

The basic unit of area was the garden-plot, which was 1 square rod or about 36 sq m, so this particular design would have been 225 garden-plots (of course, it was just a diagram on a small clay tablet!).

Symmetry:

With all the lines drawn in, the shape has 4 lines of symmetry (vertical, horizontal and the two diagonals), the same as a square.

It has rotational symmetry of order 4 (the same as a square).

There are 8 small corner triangles, another 4 immediately around the square, then another 4 composed of two small corner triangles, so 16 in total.

There are 4 small squares in the centre, and 1 larger square which they make up, then another square around this (tilted) and finally the outer square, so 7 in total.

The left-hand motif has 1 line of symmetry (horizontal) and no rotational symmetry (so order 0).

The middle motif has 2 lines of symmetry (vertical and horizontal) and rotational symmetry order 2, equivalent to a rectangle.

The right-hand motif has 4 lines of symmetry (vertical, horizontal and the two diagonals) and rotational symmetry order 4, equivalent to a square.

Numbers in base 60

1. 1 05 = 65
2. 1 47 = 107
3. 2 16 = 136
4. 3 55 = 235 (also useful to think of this as 240 - 5, or 5 less than 4 00 in base 60)
5. 5 08 = 368
6. 10 01 = 601

Tabulating the number of 3600s, 60s and then units in a base 10 number is probably the easiest way of converting from base 10 to base 60. A useful check is then to convert the number back into base 10 by multiplying the number in each column by 3600 or 60 and then adding the units.

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Number	3600s (60 x 60)	60s	Units
70	0	1	10
89	0	1	29
115	0	1	55
141	0	2	21
200	0	3	20
347	0	5	47
639	0	10	39
1001	0	16	41
3793	1	3	13

1. 999
2. 59 59 59 = 215,999
3. 987
4. 59 58 57 = 215,937

Babylonian Fractions

1. 0;20 = 20/60 = 1/3
2. 0;45 = 45/60 = 3/4
3. 0;10 = 10/60 = 1/6
4. 0;36 = 36/60 = 3/5
5. 0;55 = 55/60 = 11/12
6. 1;12 = 1 24/60 = 1 2/5
7. 2;50 = 2 50/60 = 2 5/6
8. 8;18 = 8 18/60 = 8 3/10

1. 13/30 = 26/60 = 0;26
2. 1/4 = 15/60 = 0;15
3. 2/3 = 40/60 = 0;40
4. 1/5 = 12/60 = 0;12
5. 1 3/10 = 1 18/60 = 1;18
6. 2 7/12 = 2 35/60 = 2;35

Any base 10 fraction which has a denominator which cannot be expressed as a factor of 60, 3600, ... cannot be expressed in the Babylonian way, because their only denominators were 60, 3600, and so on. So 1/8 = 450/3600 can be expressed in their notation (although it would not have been 0 450, but 0 7 30), but 1/7 cannot be expressed in their notation. Other examples are 1/11, 1/13, 1/14, and so on as 11, 13 and 14 all have factors which are not factors of 60.

Multiplication - the Babylonian way


reads 12 times 4, so the answer is 48.

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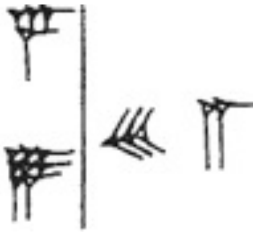

reads 4 times 8 equals 32, the two numbers to be multiplied are placed directly underneath each other, with the answer to the right of the vertical line.

1 45 = 105, 3 00 = 180, 1 30 = 90, 2 30 = 150, 1 15 = 75, 3 15 = 195

These are all in the 3, 5, and15 times tables.

In base 60, the 12 times table is 12, 24, 36 48, 1 00, 1 12, 1 24, 1 36, 1 48, 2 00, and so on. It is one of the easier ones for us in base 60, since 12 is a factor of 60.

You could do any multiplication table in base 60, since you can represent all whole numbers from 1 upwards in base 60. It is straightforward to keep adding a given number, the only problem is to remember to change place in the number after 60, and then to remember that after 120 we need to change to 2 lots of 60, and so on.

Division - the Babylonian way

1. 1/2
2. 1/5
3. 3
4. 2
5. 4/3 = 1 1/3
6. 0.3 = 3/10, so 10/3 or 3 1/3 (better than $3.\bar{3}$ in my opinion, even though the question involved a decimal, but you may disagree!)
7. 3/5
8. 2/5

Multiplying a number by its inverse gives 1, as that's the definition of a multiplicative inverse!

Base 60 inverses:

Number (base 60)	Number (base 10)	Inverse (base 10)	Inverse (base 60)	Number (base 60)	Number (base 10)	Inverse (base 10)	Inverse (base 60)
0 30	1/2	2	02	0 20	1/3	3	03
30	30	1/30	0 02	0 04	1/15	15	15
1 00	60	1/60	0 01	01 30	1 1/2	2/3	0 40

Any fraction which does not exist in Babylonian base 60 notation cannot be an inverse, so since 1/7 can't be converted in this notation, 7 does not have an exact inverse in base 60. The Babylonians did use good approximations, however.

See Babylonian Fractions above for more on this.

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Deciphering a Babylonian Tablet

The numbers on the left-hand tablet are:

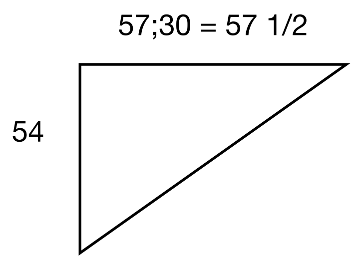
5 15 = 315

5 15 = 315

27 33 45 = 99,225

315 x 315 = 99,225 so this is correct!

The numbers on the right-hand tablet are:



Inside the triangle is:

25 52 30 = 25 x 60 + 52.5 = 1552.5

So the problem is to calculate the area of the triangle, and it is also correct.