

Epidemics: Modelling with mathematics



Teacher Notes

Navigating the Pack

This pack is intended to introduce 12-14 year-old students (Years 8 - 10 in the UK) to the concept of mathematical modelling. Mathematical models are created for a purpose - to help us understand complex processes better, and to enable us to make predictions which help determine policy and its implementation.









The context here is the spread of infectious diseases, such as measles, flu or the 2009 H1N1 (swine flu) outbreak.

Below is the list as on the webpage for this pack. This suggests an order in which the resources might be used. Any resources can, of course, be used independently of the rest, but students will gain more from the activities if they have seen the preceding video clips first.






Some of the resources in this pack are also included in the pack *Life-saving maths: How does vaccination work?*

* indicates a resource suitable for almost all students

*** indicates more challenging resources

Type of Resource	Resource Name	Difficulty	Notes
	Teacher notes and worksheet answers		Start here!
	Modelling with Mathematics: Introduction	*	Introductory video clip (3 min 2 secs)
	Standing Disease	*	Activity: The simplest of models - what does it tell us that's helpful, how could it be improved?
	Counter Plague	*	Activity: Developing the model - how is this model better, how could it be improved?
	Counter Plague: demonstration	*	Video clip (3 min 47 secs) - you could omit this if the activity instructions are clear without it, or use it as a teacher resource.
	Models of Epidemics	**	Video clip (3 mins 10 secs)
	Graphs of epidemics	**	The graphs Julia Gog talks about in Models of Epidemics, plus questions for class discussion.
	Refining the Models	**	Video clip (4 mins 30 secs)

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Type of Resource	Resource Name	Difficulty	Notes
	26-Card Disease	**	Activity: Refining the model to take account of immunity (natural or induced through vaccination)
	26-Card Disease	**	Video clip (7 mins 37 secs) - you could omit this if the activity instructions are clear without it, or use it as a teacher resource.
	Why do epidemics terminate?	***	Video clip (2 mins 42 secs) - Dr Julia Gog
	Understanding Counter Plague	***	Presentation: screenshots from e-Counter Plague to help students interpret the model, and to understand what we might expect to happen, given particular dice settings. Students should have tried the activity Counter Plague first.
	e-Counter Plague	***	Activity: a set of 4 online simulations to help investigate variations on Counter Plague. Students should have tried the activity Counter Plague first. It would also be helpful if they worked through the presentation, Understanding Counter Plague , as preparation for investigating these animations.

Standing Disease

This is a very simple model for an epidemic, in which one student chooses two others, they each choose two others, and so on. The number of people 'infected' at each stage is a power of 2 - 1, 2, 4, 8, ...

In general, it would take 4 to 6 steps for a whole class to be infected - $2^4 = 16$; $2^5 = 32$; $2^6 = 64$.

If 3 people are infected at each stage, the sequence is the powers of 3 - 1, 3, 9, 27, ..., and for n people, it is the powers of n - n, n^2, n^3, \dots

The world population is about 6.9 billion (2011). If 2 people are infected at each step, then it only takes 33 steps (counting the first infection as step 0) for the entire world population to be infected. If the infection rate is 3, then it only takes 21 steps.

Clearly real epidemics don't follow this model after the first few steps, where exponential increase might well be seen. Reasons include: this model does not take into account people who recover and are then immune, or people who were immune in the first place, or isolated communities.

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Counter Plague

This model uses a die to introduce variability into the model. The number of people infected at each step in an epidemic is determined by a die, which means we can have an infection rate which isn't a simple whole number.

There is an additional video on the activity itself, which could be used in the classroom, or as part of teacher preparation.

Two scenarios are suggested for investigation:

Number on die	Number of people infected
1	0
2	0
3	1
4	1
5	2
6	3

Number on die	Number of people infected
1	0
2	0
3	1
4	1
5	1
6	2

Given that each of the outcomes on the die are equally likely, the expected rate of infection for the left-hand table is:

$$\frac{1}{6} \times 0 + \frac{1}{6} \times 0 + \frac{1}{6} \times 1 + \frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 = \frac{7}{6}$$

As this is > 1 , then on average, we should expect these epidemics to take off, so that students need to use all the counters available to them.

For the right-hand table, however, the expected rate of infection is:

$$\frac{1}{6} \times 0 + \frac{1}{6} \times 0 + \frac{1}{6} \times 1 + \frac{1}{6} \times 1 + \frac{1}{6} \times 1 + \frac{1}{6} \times 2 = \frac{5}{6}$$

This is < 1 , so we should expect on average that epidemics will die out quite quickly.

Graphing the results of several 'epidemics' should help students to see the difference. A bar chart is to be preferred to a line graph, since the data is discrete.

This model is better than the Standing Disease, because it allows variability in the rate of infection. Since some of the results on the die mean that no one is infected, this can be interpreted either as a sick person choosing to isolate themselves, or as contact with someone who is already immune.

Models of epidemics (video) and Graphs of epidemics (activity)

In the video clip, Dr Julia Gog discusses five significant epidemics: the Black Death in the mid-14th century in the UK, the 1906 Bombay Plague epidemic, the H1N1 (swine flu)

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epidemic of 2009 in England and Wales, seasonal flu in the late 20th and early 21st centuries in the UK, and measles from 1940 in the UK.

The activity worksheet has the same five graphs on it so that students can see the data clearly for themselves. The questions on the worksheet are intended to be used for class discussion, which could take place at the relevant point in the video or be used as follow-up to the video.

They show different styles of epidemic, illustrating that there is no one model which can be used to explain all epidemics, and that knowledge of social and other factors is important also. They will also help students to become more aware of the kinds of things they should be looking for when they see a graph, if they are to interpret it fully.

Black Death:

- The drop in population between the years 1300 and 1350 was about 1.25 million, with an increase between 1350 and 1400 of about 0.5 million. Since the population in 1300 was about 3.5 million, the drop is between 30% and 40% (it would be inappropriate to be more exact, since the population figures are themselves only very approximate) - about 1 in 3 of the population.

A comparable drop now would take the UK's population from approximately 61 million to around 40 million. The effect on social and political life was immense, as it would be now.

Bombay (Mumbai) Plague:

- The epidemic was at its height in April and May of 1906, and reached a peak of over 900 deaths.
- The rise from 100 deaths to 700 took 5 to 6 weeks, an example of exponential rise at the start of an epidemic - and it should be remembered that this graph shows the death rate. The number of cases will have been greater, because not everyone died, although the death rate was high.
- The number of deaths dropped below 100 in week 25-26, so 6 months after the onset of the epidemic.

H1N1:

- A major factor in many infectious illnesses are the school terms and holidays. Schools are great places for spreading infections, with so many people mixing closely day after day, and many also travelling on public transport morning and evening. In the UK, the 5-14 year age group was affected much more than other age groups.
- In other countries, the pattern is likely to have been different if school terms were different, or if other age groups were more affected.
- There was a later peak at the end of October (approximately 84,000 cases per week reported), but this was less than the late July peak (around 110,000 cases per week reported). By the end of the year, the epidemic was virtually over.

Seasonal flu:

- The H1N1 outbreak was quite unlike seasonal flu, which peaks in the first quarter of each year. Peaks vary, but there is a well-established annual cycle.

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- The largest outbreak so far since 1988 was in 1990, when 585 people per 100,000 sought medical help. The smallest outbreak so far was in 2006, when only 10 people per 100,000 sought medical help. The average is about 125 to 130 people per 100,000 - probably the easiest way to find an estimate without the actual data is to average the max and min, so about 280 to 290 per 100,000, then to realise that this will be an over-estimate, because there are more smaller outbreaks than this than there are larger ones. Alternatively, ignore the one big peak, and treat the largest peak as 270 in 1999, which then gives a rough average of about 130 per 100,000.

Measles:

- Measles vaccination was introduced in 1968 in the UK. Thereafter the regular peaks drop off quite abruptly, with numbers of cases reported decreasing from then on.
- The triple MMR vaccine was introduced in 1988.
- The graph does not really show what has happened in recent years, but a reduction in the number of immunisations below the 95% threshold needed to prevent epidemics occurring has meant that it is now a threat to children again. If you are interested in knowing more about this, have a look at our pack on Vaccination.

26-card Disease

You may wish to use the video that accompanies this activity as preparation, or for students to view.

This model makes explicit the effect of people recovering and can also be used to model what happens when people are immunised, if some black cards are included in the population right from the start.

Keeping the population at 26 is a way of simulating what happens in a community of a given size, rather than looking at what could happen if contacts outside the community are also considered. Even with two potential infections per infected person, the epidemic rarely affects everyone. This provides a reasonable model of what would happen in, say, a city where most people live and work in the city with few outside contacts.

The probability that no one is infected at Step 1 is 0. The first red card is replaced by one black card, but two cards are placed on the table at Step 1, so at least one of them must be red. To see what might happen subsequently, we need to find a way to record the possible outcomes.

Here are the first three steps (bearing in mind that a black card is not left on the table, but replaced in the population pack before moving on to the next step):

	B available	Possible outcomes
Step 0	0	R
Step 1	1	RR R(B)

Outcome at Step 1	B available	Possible outcomes at Step 2
RR	3	RRRR RRR(B) RR(BB) R(BBB)
R	2	RR R(B) (BB)

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The probability that the epidemic terminates at Step 2 is given by:

$$\begin{aligned} & p(B \text{ at Step1}) \times p(BB \text{ at Step2}) \\ &= \frac{1}{26} \times \left(\frac{2}{26} \times \frac{1}{25} \right) \\ &= \frac{2}{16,900} \approx 0.000118 \end{aligned}$$

e-Counter Plague

The presentation is designed to help students understand the animations in the e-Counter Plague set.

The screen shot shows the simplest version.

Students can set (bottom left grid):

- number of susceptible green smileys
- number of initial infected red smileys
- number of initial immune blue smileys

The die can be set (top left grid) so that infections occur randomly (see Counter Plague above for details). An expected rate of infection of less than 1 should mean that on average, epidemics take off, and all the green smileys (susceptible to the disease) are gradually infected (red) then recover (blue). An expected rate of infection of less than 1 should mean that on average, epidemics do not take off, and some green smileys remain when the epidemic terminates. An expected rate of infection equal to 1 is unpredictable.

Once students have grasped the simple version, they can then look at two-class versions, which show what happens when an infection passes from one classroom to another, and perhaps back again.

The model also applies to any two communities where there are contacts between them.