

MATHS OF CHURCHES, MOSQUES, SYNAGOGUES AND TEMPLES:

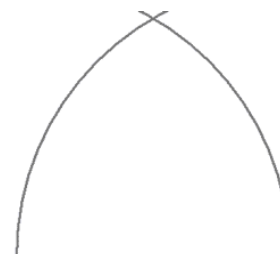
Constructing arches and windows

I suggest students are pointed towards appropriate sections. Those who want something more mathematical should answer the additional questions, those who are more interested in the design aspects could concentrate on drawing. All should ensure their drawing is accurate. If any students find it too difficult to do the constructions on plain paper, they could use isometric paper for the general arch and for the windows.

Use the accurate constructions given on a separate sheet.

1. General arch (easier)

1. Draw a light line in pencil between 5 and 10 cm long (the exact length doesn't matter).
2. Put the point of your compasses on one end of the line, and set the pencil to the distance to the other end.
3. Now draw an arc from that end of the line until you are at least half way back.
4. **Keeping the compasses set to exactly the same radius**, put the point of the compasses on the other end of the line, and then draw a second arc so that it meets the first arc.



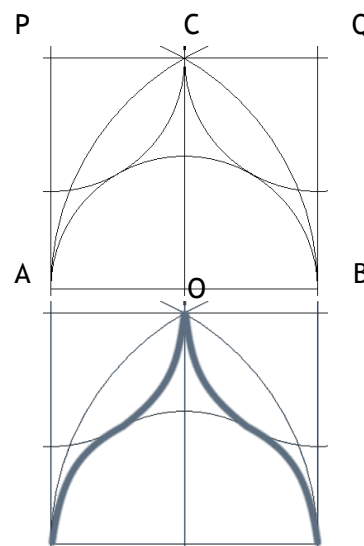
Questions to answer:

- If you drew in straight lines from the ends of the baseline to the vertex of the arch, what shape would you have? *Equilateral triangle*
- Justify your answer, using logical reasoning. Only use measurement to confirm your answer. *Eg. The radius of the arcs is equal to the length of the base, so the distance from each base vertex to the vertex of the arch is equal to the length of the base, giving a shape with three equal sides.*

2. Ogee arch (harder, f/up questions require trig)

You will need the accurate construction for perpendicular lines.

1. Using ruler and compasses, construct a semi-circle on a horizontal baseline, AB.
2. Follow the instructions for constructing a basic arch (above) to find a point, C, directly above the centre of the baseline, O.
3. Using the accurate construction method, construct three lines perpendicular to the baseline at either end and at the centre, ie. from A to P, from B to Q and from O to C. (You will need to extend your baseline in both directions to construct the outer two verticals).
4. Draw another line, perpendicular to these three, so that it forms the top of a rectangle and passes through the vertex of the arch, PCQ. (You will need to extend the line OC to do this).
5. Draw two quarter-circles centred at P and Q, with radius equal to that of the semi-circle.
6. Emphasise arcs as shown on the right to create the ogee arch.

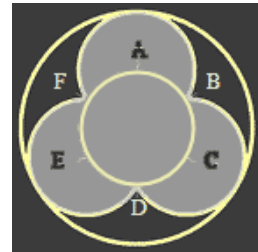


Questions to answer:

- What is the height of the arch if the radius of the semi-circle is r ? (Hint: draw in a diagonal from O to P or Q. What is its length, relative to r ?).
Adding the diagonal forms a right-angled triangle in which the base is r and the hypotenuse is $2r$. Pythagoras' Theorem then gives the height as $r\sqrt{3}$.
- Calculate the angle between OP and OA. Use this to find out the proportion of a circle of which each of the constituent arcs of the ogee arch is formed.
Angle is $\cos^{-1} \frac{1}{2} = 60^\circ$, so each arc section is $1/6$ of a complete circle.
- If the radius of the initial circle is r , what is the length of one side of the arch?
 $2 \times 1/6$ of the circumference of the circle, ie. $1/3 \times 2\pi r$

3. Trefoil Window (easier)

1. Draw a circle, and put six equally spaced points, A, B, C, D, E, F around it (see separate instructions).
2. **Keeping your compasses set to the same radius**, construct an arc of a circle, with A as centre, and B and F as the ends of the arc.
3. **Keeping your compasses set to the same radius**, repeat with C and E as centres.
4. Using the centre of the original circle, draw a circle around these three arcs to enclose them.



Questions to answer:

- What fraction of a circle are the three arcs? *Two-thirds.*
- Can you justify your answer? *Eg. arc not drawn subtends an angle of 120° , so arc subtends an angle of 240° , which is $2/3 \times 360^\circ$*
- What is the connection between the radius of the original circle and the final circle?
Double. Eg. Original circle passes through the centres of the three arcs, so radius of large circle is twice that of the original circle.

4. Rose windows

You can really use your imagination with these windows, so detailed instructions are not given - but use accurate construction techniques. You may find a circle split into four equal sectors (by drawing a diameter, then bisecting it) or a 6-point circle are good starting points, and bisecting angles is also very useful. Some examples are shown to inspire you!

