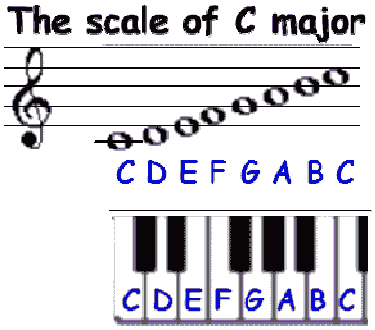
The mathematics of musical scales

- A scale is a regular sequence of notes.
- Most western music written between the 17th and 20th centuries is based on major or minor scales.
- The scale of C major uses just the white notes on a keyboard and is shown on the right.



The just scale

- Before the time of Bach in the 17th century, musicians used *just scales*.
- In a just scale, all musical intervals are based on rational frequency ratios, ie. frequency ratios which can be expressed as fractions (rational numbers).

The interval from C to G has a frequency ratio of 3/2 and the interval from C to F has a frequency ratio of 4/3. Using this information, we can work out the frequency ratio of the interval from F to G, going from F to C to G:

$$F \xrightarrow{\times\frac{3}{4}} C \xrightarrow{\times\frac{3}{2}} G$$

$$F \xrightarrow{\times\frac{9}{8}} G$$

To help you get the ratios the right way round, remember that higher notes have higher frequencies, so:

- the frequency of C is less (3/4) than that of F
- the frequency of G is more (3/2) than that of C

This means that G has a frequency 9/8 times that of F, or that the frequency ratio of G to F is 9:8. The fact that we don't know the actual frequencies doesn't matter, since we are just working with ratios.

Q1 Use the information below to work out the frequency ratios of all the notes in a just scale based on C, ie. C to D, C to E, C to F, C to G, C to A, C to B, and C to C' (C' is the upper C one octave above the first C).

- The frequency ratio of C to E (a third) is 5/4
- The frequency ratio of C to F (a fourth) is 4/3
- The frequency ratio of C to G (a fifth) is 3/2
- The frequency ratio of C to C' is 2/1
- The frequency ratio from C to D is the same as from F to G.
- The frequency ratio from C to A is the same as from D to B.
- The frequency ratio from G to B is the same as from C to E.

Q2 When you've found the frequency ratios of these intervals, try finding the ratios of adjacent notes, ie. C to D, D to E, E to F, and so on, up to B to C'.

- What do you notice?
- Can you see why black notes on a keyboard are grouped in twos and threes?
- Are all the frequency ratios of the tones the same?
- There is a big problem with the just scale – can you see what it is?